Integral Method for the Calculation of Incompressible Two-Dimensional Transitional Boundary Layers

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The method proposed here considers the mean flow in the transition zone as a linear combination of the laminar and turbulent boundary layer in proportions determined by the transitional intermittency, the component flows being calculated by appropriate integral methods. The intermittency distribution adopted takes into account the possibility of subtransitions within the zone in the presence of strong pressure gradients. A new nondimensional spot formation rate, whose value depends on the pressure gradient, is utilized to estimate the extent of the transition zone. Onset location is determined by a correlation that takes into account freestream turbulence and facility-specific residual disturbances in test data. Extensive comparisons with available experimental results in strong pressure gradients show that the proposed method performs at least as well as differential models, in many cases better, and is always faster.

Nomenclature

C	= coefficient of entrainment for turbulent boundary layer
C_f	= skin-friction coefficient
$F(\gamma)$	$= [-\ln(1-\gamma)]^{1/2}$
H	= boundary-layer shape factor, $=\delta^*/\Theta$
h	= power law index for turbulent flow
K	= pressure gradient parameter, $\equiv(\nu/U^2)\,\mathrm{d}U/\mathrm{d}x$
k	= turbulent kinetic energy
L	= pressure gradient parameter, $=(\Theta_L^2/\nu)\,\mathrm{d}U/\mathrm{d}x$
N	= nondimensional spot formation rate
$N_0(q)$	= value of N at given q for $L=0$
n	= spot formation rate per unit time and spanwise distance
P^*	= velocity profile factor [see Eq. (16)]
Pr	= Prandtl number
q	= freestream turbulence intensity, %
q_0	= equivalent freestream turbulence for residual disturbances associated with a given facility
Re_Θ	= Reynolds number based on momentum thickness
$Re_{\Theta T}$	$= Re_\Theta$ based on Θ_T
St	= Stanton number
T_w/T_e	= wall-to-freestream temperature ratio
$U(x)$	= freestream velocity
u	= nondimensional velocity in boundary layer
X_u	= unheated starting length
x	= streamwise coordinate
x^*	= streamwise location of "kink" in $F(\gamma)$ plot
$x$step	= station at which lag-entrainment scheme is initiated
y	= coordinate normal to surface
γ	= transitional intermittency
δ	= boundary-layer thickness
δ^*	= boundary-layer displacement thickness
η	$= y/\delta$
Θ	= boundary-layer momentum thickness
λ	= transition zone length, $= x(\gamma=0.75) - x(\gamma=0.25)$
ν	= kinematic viscosity
ξ	$=(x-x_t)/\lambda$
σ	= nondimensional spot propagation parameter in constant pressure

Subscripts

L, T	= laminar and turbulent values, respectively
S	= location of $\gamma=0$
t	= onset location
1,2	= first and virtual onset locations

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Introduction

THE computation of many practical boundary-layer flows is often critically dependent on the modeling of the laminar-turbulent transition zone. Indeed, recent progress in numerical modeling of turbulent flows for technological applications has reached a stage where "perhaps the most important immediate modeling problem is that associated with the representation of transition."[1] This is particularly so in applications involving relatively low Reynolds numbers or extensive transitional zones, such as remotely piloted vehicles, turbomachine blades,[2-5] the Space Shuttle,[6] etc. For engineering analysis and design, where computational economy and accuracy in a specific class of flows is important, it seems worthwhile to develop, if possible, a simple integral scheme for the prediction of transitional parameters embedded in a comprehensive method covering laminar and turbulent regions as well. The development of such a scheme for two-dimensional flows in strong pressure gradients forms the theme of this paper.

Available transition zone models may be classified into four groups.[7] The first of these treats the mean flow during transition as a linear combination in the proportions $(1-\gamma):\gamma$, where γ is the transitional intermittency of the mean flow in the laminar boundary layer starting from the stagnation point and of the turbulent boundary layer starting from an appropriate onset station x_t. For example, the mean velocity and skin friction are, respectively, given[8] by

$$u=(1-\gamma)u_L+\gamma u_T \tag{1}$$

$$C_f=(1-\gamma)C_{fL}+\gamma C_{fT} \tag{2}$$

Here (and in what follows) subscripts L and T denote values for the laminar and turbulent layer, respectively, each starting from its respective origin, and u is nondimensionalized with respect to the freestream velocity $U(x)$. It has been shown[8] that this model gives an excellent description of mean velocity profiles, skin friction, and all integral parameters in the transition zone in constant pressure flows. A few attempts to extend the model to more complex situations have been made[9] but have not yet been validated against recent experimental data.[10-12] The model used by Chen and Thyson[13] adopts an intermittency distribution that does not agree with more recent experiments.[14]

Arnal[15] uses an intermittency distribution that is specified in terms of the momentum thickness Θ and is not derivable from the spot theory of transition. Furthermore, Arnal not only predicts C_f using Eq. (2) but also the shape parameter H by a similar linear combination relation,

$$H = (1-\gamma)H_L + \gamma H_T \tag{3}$$

However, as Θ is itself a nonlinear function of the intermittency (see Eq. 5 below), it cannot be obtained from a linear combination of the type in Eq. (2), therefore H cannot be either.

Several algebraic[16-21] and differential[22-26] models have been proposed for transitional flows. The former generally use an eddy viscosity that is gradually turned on during the transition zone in a manner determined by the intermittency. Differential models may use one[22] or two[23-26] additional equations for closure but often require considerable tuning[24,27] to obtain reasonable results. Since this work was completed we have become aware of the work of Fraser and Milne[28] that proposes an integral scheme similar in some ways to the present model. However, none of the available models has used recent developments in understanding intermittency distributions,[10] in particular, the possibility of subtransitions[29] in highly favorable pressure gradients and in the specification of spot formation rates.[30] Furthermore, comparisons with recent experimental data[10-12] are very limited[20,25] and, in particular, do not include the measurements showing subtransitions reported in Ref. 10. It is our objective here to show that, if these developments are taken into account, it is feasible to develop a simple linear-combination-type method that compares favorably with experiment. It is well known[31] that properly formulated integral methods for turbulent boundary-layer calculations can be at least as good as more complex field methods and can in any case provide complementary capabilities especially in engineering design.

Present Approach

As a consequence of the linear combination principle in Eq. (1), such parameters as displacement and momentum thickness and shape parameter can be readily estimated from the easily derived relations

$$\delta^* = (1-\gamma)\delta_L^* + \gamma\delta_T^* \tag{4}$$

$$\Theta = \gamma(1-\gamma)\int_0^\delta [u_L(1-u_T) + u_T(1-u_L)]\,\mathrm{d}y + (1-\gamma)^2\Theta_L + \gamma^2\Theta_T \tag{5}$$

where the boundary-layer thickness δ in Eq. (5) is given by

$$\delta = \delta_L \qquad \text{if } \delta_L > \delta_T \tag{6a}$$

$$= \delta_T \qquad \text{if } \delta_T > \delta_L \tag{6b}$$

In addition, the skin-friction coefficient is estimated from the relation in Eq. (2). Note that Eq. (5) rules out a linear-combination expression for Θ.

The physical basis of the method proposed here has been described at length in some of the authors' previous publications.[7,10,29,30] It will only be sketched here together with the structure of the computer code TRANZ 2 that was written to implement the method. Briefly, the idea is to construct a linear-combination-type model that computes the laminar boundary layer starting from the leading edge using a modified Thwaites method[32,33] and the turbulent boundary layer from an appropriate onset location using the lag-entrainment method.[34] These component models have been widely tested and possess well-understood advantages. The boundary layer in the transition zone is then calculated using Eqs. (2) and (4–6). Referring to Fig. 1, the steps in the method and the associated modules in the code (each to be discussed subsequently) are as follows.

1) Based on the modified Thwaites scheme, which will be described later, the module LAMFLO provides δ_L, δ_L^*, Θ_L, C_{fL}, and the pressure gradient parameter $L(=\Theta_L^2 U'/\nu$, where ν is the kinematic viscosity; $U' = \mathrm{d}U/\mathrm{d}x$). A laminar boundary-layer velocity profile $u_L(y)$ is provided for each value of L by the module LAMVEL.

2) Based on the lag-entrainment scheme, the turbulent parameters δ_T, δ_T^*, Θ_T, and C_{fT} are provided by the module TURFLO. The module TURVEL provides the turbulent velocity profile $u_T(y)$.

3) Prediction of transition onset is based on the correlation proposed recently[35] in terms of the the freestream turbulence intensity and pressure gradient. However, provision is also made for specifying the onset location independently, if necessary.

4) The transition zone length is based on estimates of the spot formation rate, provided by the module EXTENT.

5) The intermittency distribution is obtained from the module INTER, based on parameters from ONSET and EXTENT.

6) The transitional parameters are finally derived from Eqs. (2) and (4–6) using the outputs of the above modules.

Onset of Transition

The prediction of onset location is still an open problem and cannot be attempted at present except through empirical correlations of one type or another. In the present method it is recognized that the onset location depends strongly on not only the pressure gradient but the disturbance environment, which includes freestream turbulence q and, in the case of data obtained in test facilities, other residual nonturbulent disturbances like noise, vibration, etc. (The onset location referred to here generally corresponds to the origin of the emerging turbulent boundary layer.) Recently,[35] it has been shown that the residual disturbances can be parameterized by an equivalent freestream turbulence q_0 and that a useful correlation for the onset Reynolds number is

$$Re_{\Theta t} = 0.9 Re_{\Theta t0}[1 + 0.15(e^{-q} + 2)(1 - e^{-60L})] \tag{7}$$

where

$$Re_{\Theta t0} = 100 + 310/(q^2 + q_0^2)^{1/2} \tag{8}$$

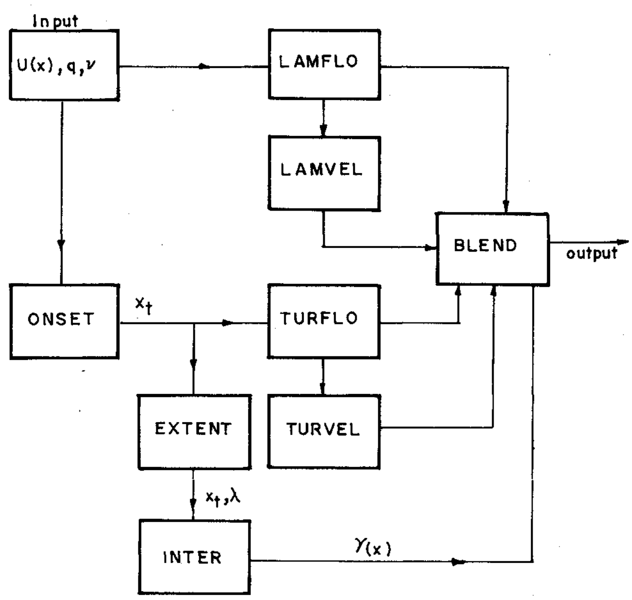

Fig. 1 Schematic structure of the present model.

is clearly the value at zero pressure gradient ($L=0$), $q=100(2k/3)^{1/2}/U$ is the freestream turbulence intensity in terms of the turbulent kinetic energy, and q_0 is a number to be determined for each facility (and possibly for each test condition). Equations (7) and (8) are used in the present model because they show the best agreement with available experimental data among the published correlations.

Intermittency Distribution

Of the various intermittency distributions proposed,[13,36,37] the universal nature of the constant pressure γ distribution[36]

$$\gamma = 1-\exp(-0.41\,\xi^2), \qquad \xi=(x-x_t)/\lambda \tag{9}$$

$$\lambda = x(\gamma=0.75)-x(\gamma=0.25) \tag{10}$$

has been confirmed,[7,8,10,38,39] even at hypersonic Mach numbers.[40] It is important to note that, when intermittency data are available, the best choice of the onset location x_t in Eq. (9) is made by extrapolating the best-fit linear variation of $F(\gamma)=[-\ln(1-\gamma)]^{1/2}$ with x to $F(\gamma)=0$, as this location has also been found to represent the effective origin of the turbulent boundary layer that emerges after transition. However, small nonzero intermittency values at x_t, so determined, are not unusual, especially at low Reynolds numbers because under such conditions spot growth may go through an initial nonlinear region of the kind that can be seen in the experiments of Schubauer and Klebanoff[41] with artificially generated spots.

In certain classes of pressure gradients, especially those that are stabilizing near onset, subtransitions can occur. Equation (9) then holds in segments[42]:

$$\gamma = 1-\exp(-0.41\,\xi_1^2),\quad \xi_1=(x-x_{t1})/\lambda_1, \quad \text{for } x\le x^* \tag{11}$$

$$\gamma = 1-\exp(-0.41\,\xi_2^2),\quad \xi_2=(x-x_{t2})/\lambda_2, \quad \text{for } x\ge x^* \tag{12}$$

where x_{t1}, x_{t2}, λ_1, and λ_2 denote various streamwise locations and lengths illustrated in Fig. 2, which shows intermittency data from one of the experiments reported in Ref. 7; a favorable pressure gradient was applied near the onset of transition in this flow; and x^* denotes the location of the kink in the $F(\gamma)$ plot. Such a kink is evidence of what is called a subtransition[29] and is probably caused by a relatively sudden change in the nature of the flow from subcritical to supercritical due to change in flow stability. Interestingly, the available data on boundary-layer thickness in favorable pressure gradients near onset indicate that the streamwise location x_{t2} represents the origin of the turbulent boundary layer that emerges after transition, as may be seen from Fig. 3. In Fig. 3, flows DFU1 and DFD1 of Narasimha et al.[10] are shown along with the flow (to be called BW22 here) reported by Blair and Werle[11] at the conditions at $K=0.2/10^6$ with grid −2 and $q=2\%$. For specification of the γ distribution in Eq. (12), the more important zone length, therefore, is also λ_2.

The intermittency distribution in Eq. (9) [with Eq. (12) for pressure gradients] is adopted in the present model.

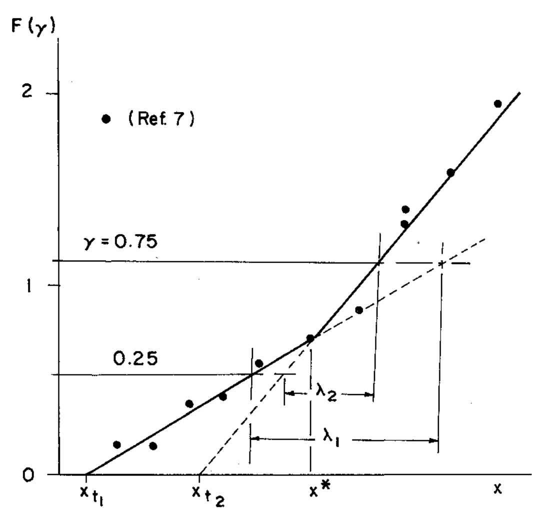

Fig. 2 Definitions of x_{t1}, x_{t2}, x^*, and two length scales λ_1 and λ_2, based on $F(\gamma)$ vs x plot (data NFU1 of Ref. 7).

Extent of Transition

In the present model, the zone length λ (or λ_2 if there are subtransitions) is obtained utilizing recent proposals[30,42] on the spot formation rate n (per unit spanwise distance and unit time). These proposals use the nondimensional spot formation rate

$$N=n\sigma\Theta_t^3/\nu$$

where σ is the Emmons spot propagation parameter[7] and Θ_t is the momentum thickness at x_t. In constant pressure flows, N has a value of about 0.7×10^{-3} in turbulence-driven transition.[30] In favorable pressure gradients, the relevant nondimensional spot formation parameter N_2 ($=n\sigma_2\Theta_{t2}^3/\nu$, where σ_2 is the spot propagation parameter in the region $x\ge x^*$ and Θ_{t2} is the momentum thickness at x_{t2}) increases with pressure gradient following the relation[42]

$$N_2(q)-N_0(q)\cong 0.25L_{t2}^2 \qquad L_{t2}=\Theta_{t2}^2U'/\nu \tag{13}$$

Here, $N_0(q)$ is the value of N at the same freestream turbulence level without pressure gradient, and L_{t2} is the value of L based on conditions at x_{t2}. The zone lengths λ and λ_2 are estimated, respectively, from the relations

$$\lambda=[0.41U\Theta_t^3/N\nu]^{1/2}$$

$$\lambda_2=[0.41U(x_{t2})\Theta_{t2}^3/N_2\nu]^{1/2} \tag{14}$$

Estimation of Laminar and Turbulent Parameters

Figure 4 shows a schematic representation of various computational domains in the present scheme.

The laminar parameters are estimated, for all $x>0$, using an extended and modified version of the Thwaites[32] method proposed recently[33] to take account of highly accelerated flows

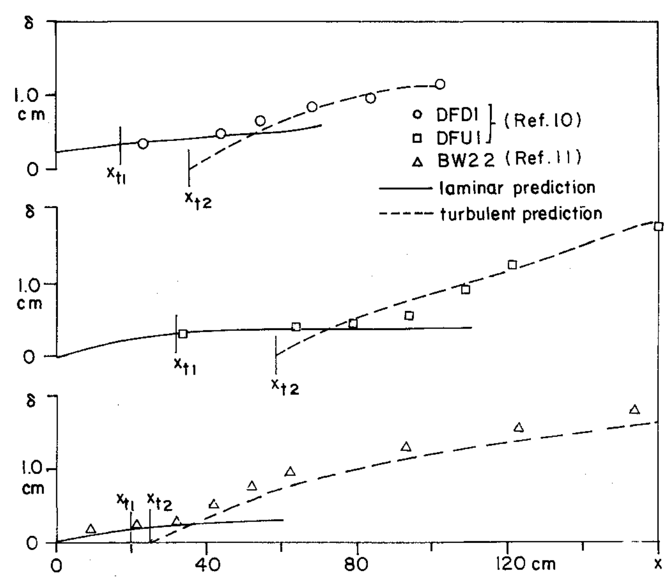

Fig. 3 Virtual origin (x_{t2}) of the turbulent boundary layer as inferred from the x variation of the boundary-layer thickness.

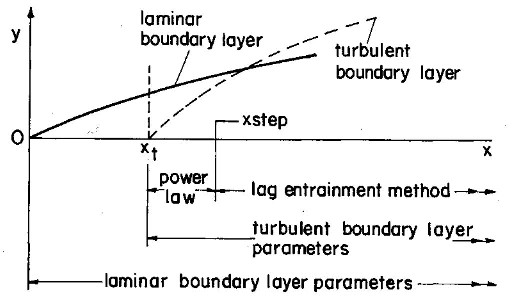

Fig. 4 Various computational zones adopted in the present model.

and to provide certain additional parameters needed in the present transition zone model. Thus, Θ_L is obtained from the relation[33]

$$\Theta_L^2 = \left(\frac{0.45\nu}{U^6}\right)\left[\int_0^x U^5\,\mathrm{d}x + 0.9\int_0^x \left(\frac{U'^2}{U^7}\right)\left(\int_0^x U^5\,\mathrm{d}x\right)^2 \mathrm{d}x\right] \quad (15)$$

The first term on the right represents the Thwaites value; the second term, usually small, provides a useful correction at high favorable pressure gradients (in the range $0.25 \leq L \leq 0.4$). Similarly, the quartic velocity profile[33]

$$u_L = 2\eta_L - 2\eta_L^3 + \eta_L^4 + P^*\eta_L(1-\eta_L)^3 \qquad \eta_L = y/\delta_L \quad (16)$$

is adopted for evaluation of Θ in Eq. (5); here P^* does not correspond to the Pohlhausen[43] pressure gradient parameter but is taken as a velocity profile factor so selected that the profile in Eq. (16) gives a good representation of the Falkner-Skan solutions.

The turbulent parameters are estimated, for $x \geq x_t$ or x_{t2}, using the lag-entrainment scheme of Green et al.,[34] which is an improvement upon an earlier version based on Head's[44] method and introduces an equation for the streamwise rate of change of the entrainment coefficient $C\{\equiv U^{-1}\mathrm{d}[U(\delta_T - \delta_T^*)]/\mathrm{d}x\}$. The lag-entrainment method has an accuracy comparable[34] to that of the best of the differential methods assessed by the Evaluation Committee in the 1968 Air Force Office of Scientific Research IEP-Stanford Conference on turbulent flows[31] and is also quite successful in compressible pressure gradient flows.[34,45] The initial conditions for the governing equations are obtained using a power law profile $u_T = (y/\delta_T)^{1/h}$ in the region $x_t \leq x \leq x\mathrm{step}$ (see Fig. 4); the value of h is taken as 3.4, based on an analysis of the constant pressure low Reynolds number measurements of Purtell et al.[46] The parameters δ_T and C_{fT} are adjusted to match the values from the method of Green et al.[34] at $x$step. For $x \geq x$step, the turbulent velocity profile for use in Eq. (5) is the log plus wake profile.[31]

Results and Discussion

It can be stated at the outset that not many experiments provide suitable test cases for validation of the present model, mainly because simultaneous measurements of both intermittency and boundary-layer parameters are rare in nonzero pressure gradients and measurements of skin friction are almost entirely absent. Although we have considered[47] all of the experimental data available,[10-12,37,41] there is space only for presenting a few illustrative comparisons.

We may note here that the present linear-combination model does not require a specific "end" of the transition zone (such as considered in Ref. 37), as Θ, δ^*, H, and C_f attain their respective fully turbulent values with γ approaching unity asymptotically. It also may be noted that, unless explicitly stated otherwise, both x_t (or x_{t2}) and λ (or λ_2) are obtained, respectively, from the ONSET and EXTENT modules already described.

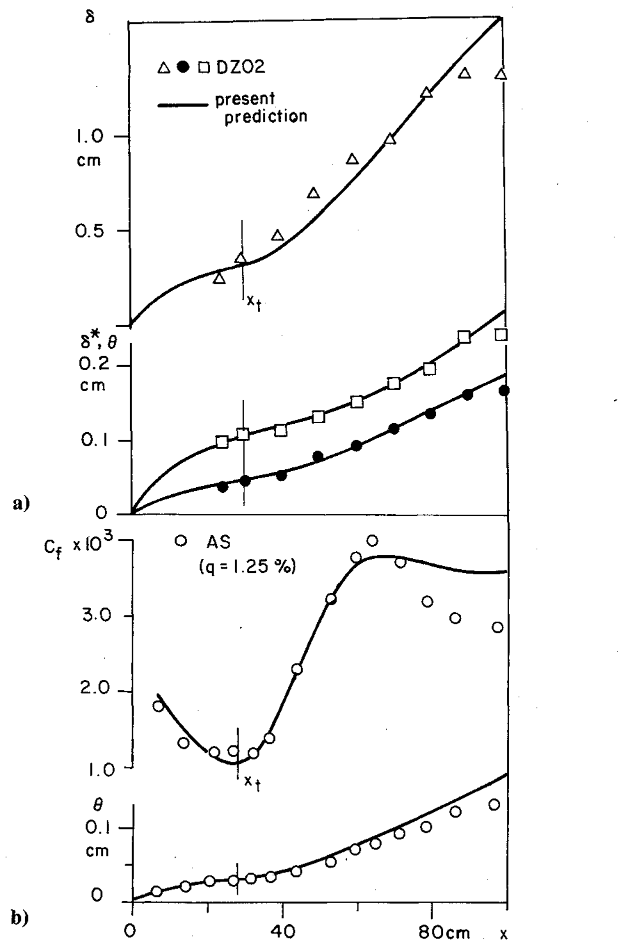

Fig. 5 Comparison of the data of Narasimha et al.[10] (DZ02) and Abu-Ghannam and Shaw[37] (AS) with present predictions.

Zero Pressure Gradient

The boundary-layer parameters predicted for the flow DZ02 (at $U = 11.54$ m/s) of Narasimha et al.[10] and that reported by Abu-Ghannam and Shaw[37] at $U = 22$ m/s and $q = 1.25$ (hereafter referred to as AS) are compared with the measurements in Fig. 5, where the onset locations $x_t (= 30$ cm for DZ02 and 28 cm for AS) are also marked. (Similar locations for constant pressure data, and x_{t1}, x_{t2} for pressure gradient data, will always be marked in what follows.) Figure 5a shows that the present model is successful in predicting the transitional parameters in flow DZ02. It is seen in Fig. 5b that agreement is generally good for $x \leq 70$ cm for the AS data. The largest discrepancy (of 27% in C_f) seen here is mainly because of the fact that the C_f data in what AS identify as the fully turbulent region are appreciably lower[14] than those given by the correlation of Green et al.[34]; however, the correlation is in excellent agreement[14] with the low Reynolds number constant pressure measurements of Purtell et al.[46] It may be noted that the onset location given and adopted by the present scheme ($x_t = 28$ cm) differs from the number quoted by Abu-Ghannam and Shaw, $x_S \approx 33$ cm, where they consider x_S as corresponding to $\gamma = 0$. This difference is due to the difference in the definitions of the beginning of transition adopted by these authors and in the present analysis.[30]

Blair and Werle[12] (who have measured heat-transfer rates as well as various boundary-layer parameters) compare their experimental data with predictions based on the turbulence models of McDonald and Kreskovsky[48] and McDonald and Fish,[22] but, in general, prefer the former (which will be compared with present predictions in the following). Both models, however, fail to predict the measured distribution of Stanton number St in the flow (hereafter referred to as BW00) at

$U = 30.48$ m/s and $q = 0.25\%$. On the other hand, we find that the simple linear-combination relation

$$St = (1-\gamma)St_L + \gamma St_T \tag{17}$$

where the γ distribution is given by Eq. (9), adequately predicts the heat-transfer distribution in the transition zone, as shown below. The St_L in Eq. (17) is estimated utilizing the relation[12]

$$St_L Pr^{0.666} = 0.453(Ux/\nu)^{-0.5}[1-(X_u/x)^{0.75}]^{-0.333} \tag{18}$$

where X_u denotes the unheated starting length (taken[12] as 4.3 cm), and the Prandtl number Pr (taken as 0.72 in the present analysis). Blair and Werle[12] suggest that St_T can be estimated using the correlation

$$St_T Pr^{0.4} = 0.0307(Ux/\nu)^{-0.2}(T_w/T_e)^{-0.4} \tag{19}$$

where T_w/T_e is wall-to-freestream temperature ratio. Noting that the turbulent boundary layer has its origin at x_t and that the streamwise length of flow development for the turbulent parameters is, therefore, $(x-x_t)$, St_T is estimated here using $(x-x_t)$ in place of x in Eq. (19). Figure 6 shows the comparison of the present prediction with the experimental data and the prediction of Blair and Werle[12]; the value of $Re_{xt}(=Ux_t/\nu)$ is also indicated in the figure. It can be seen that

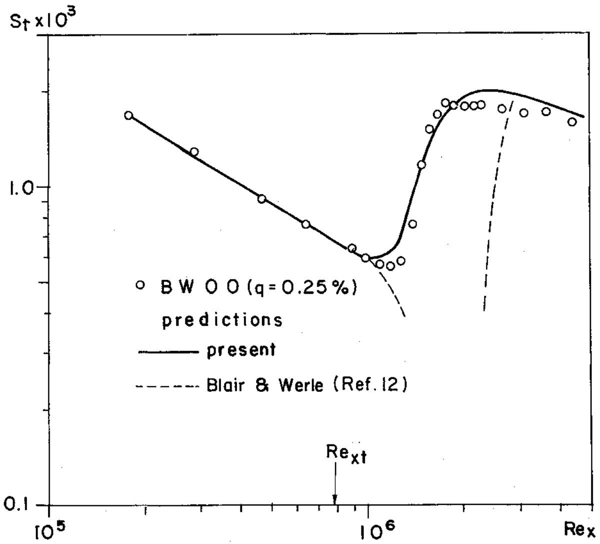

Fig. 6 Comparison of various predictions of Stanton number with measurements in flow BW00 of Blair and Werle.[12]

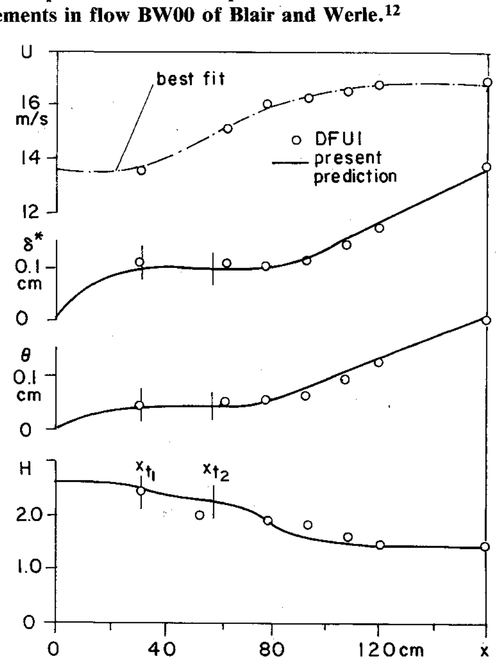

Fig. 7 Comparison of the present predictions for δ^*, Θ, and H with the measurements in flow DFU1 of Ref. 10.

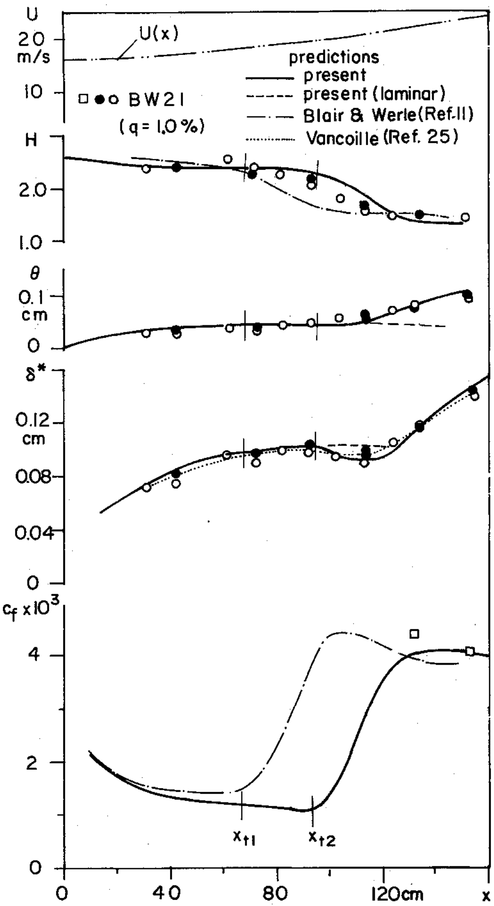

Fig. 8 Comparison of various predictions with measurements in flow BW21 of Blair and Werle.[11]

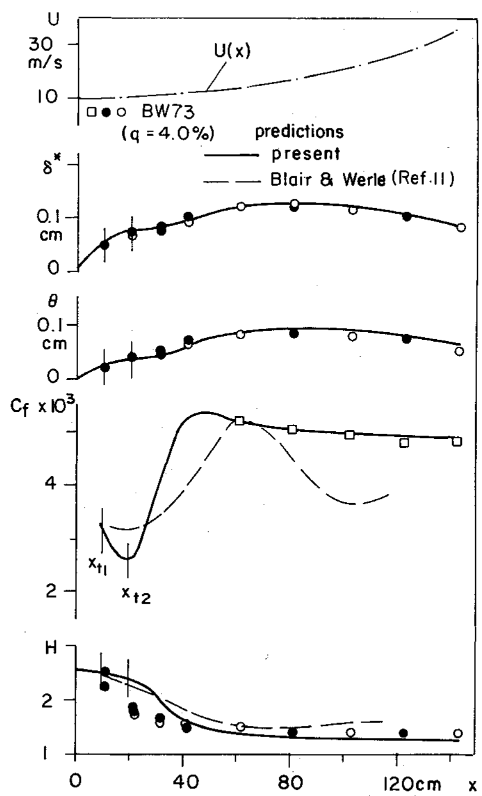

Fig. 9 Comparison of various predictions with measurements in flow BW73 of Blair and Werle.[11]

the present linear-combination model is in better agreement with the data.

Agreement is also found[47] between the present predictions for heat-transfer and integral parameters and the measurements of Blair and Werle[12] at $q = 1$ and 2%, but the comparisons are not reported here for lack of space.

Pressure Gradient

Figure 7 compares the present prediction of boundary-layer parameters with measurements in flow DFUI of Ref. 10, which has a favorable pressure gradient applied near the onset; the value of L_{t2} is 0.07. The γ distribution is available from experiment for this flow. The freestream velocity distribution is also shown in the figure. It is seen that the present predictions for δ^*, Θ, and H are in good agreement with the experimental data. It can also be seen that in the region $x_{t1} \leq x \leq x_{t2}$, the fully laminar calculation is itself satisfactory, mainly because the intermittency in this region is very low (<0.1).

Blair and Werle's[11] datasets analyzed here are flows with favorable pressure gradients at all stations, the first at $K = 0.2/10^6$ with grid 1 and the second at $K = 0.75/10^6$ with grid 3 (referred to as BW21 and BW73, respectively, in the following); $q = 1$ and 4%, respectively. Because a favorable pressure gradient applied near the onset of transition can result in a relatively longer transition zone,[10] the extent of transition, in particular the region $x_{t1} \leq x \leq x_{t2}$ if there is a subtransition, will tend to be quite long in these flows although the intermittency in the region will be small. It may be noted that the limited C_f data (2–5 points) of Blair and Werle[11] are all toward the end of transition. Further, the scales for the ordinates indicated in Fig. 49 of Blair and Werle[11] need to be shifted[49] by -0.4 in order to be consistent with their Figs. 37, 39, 42, and 44; this correction has been incorporated below. The present predictions of various boundary-layer parameters for flows BW21 and BW73 are compared with the experimental data and predictions by Blair and Werle[11] in Figs. 8 and 9; however, they do not report any predictions for Θ and δ^*. The predictions of Vancoillie[25] are also shown in Fig. 8. (It may be noted that the scales for the abscissa indicated in Fig. 6 of Vancoillie[25] need to be multiplied by a factor of 2 in order to be consistent with the data for BW21; this correction is incorporated here.) As will be seen from Figs. 8 and 9, the present calculations indicate the presence of subtransitions in both flows at the stations shown; it is gratifying that more recent intermittency measurements[49] do indeed reveal the subtransitions inferred here. Figure 8 clearly indicates that the present model is as good as the differential model of Vancoillie[25]; the small difference seen for δ^* is mainly due to the enlarged scale used compared to that for Θ. It can be seen from Figs. 8 and 9 that the present predictions show good agreement with the experimental data. Note in particular the predicted dip in δ^* for BW21. The laminar calculations for δ^* and θ in flow BW21, for which the region $x_{t1} \leq x \leq x_{t2}$ is large, clearly indicate that the growth of the turbulent spots in this region must be negligible. Comparing the present predictions for H and C_f with those of Blair and Werle,[11] it is seen that, in general, the present integral method performs at least as well as the differential model used by Blair and Werle and does better for C_f in the flows BW21 and BW73.

Although only favorable pressure gradient results are presented here, it is expected that the present model should remain useful in adverse pressure gradients with the incorporation of the proposal of Gostelow[50] on the spot formation rates in such flows.

Conclusion

A linear-combination-type integral model for the calculation of the transitional boundary-layer parameters in two-dimensional incompressible flows is proposed here. The model is found to be nowhere inferior in performance to more time-consuming differential models; in certain cases it is appreciably superior, and is always faster (and has, in fact, been implemented on a personal computer).

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References

[1]Cebeci, T., Stewartson, K., and Whitelaw, J. H., "Calculation of Two Dimensional Flow Past Airfoils," *Proceedings of the Second Symposium on Numerical and Physical Aspects of Aerodynamic Flows,* edited by T. Cebeci, Springer Verlag, New York, 1983, pp. 1–40.

[2]Turner, A. B., "Local Heat Transfer Measurements on a Gas Turbine Blade," *Journal of Mechanical Engineering Science,* Vol. 13, No. 1, 1971, pp. 1–12.

[3]Evans, R. L., "Boundary-Layer Development on an Axial-Flow Compressor Stator Blade," *Journal of Engineering for Power,* Vol. 100, April 1978, pp. 287–293.

[4]Brown, A., and Burton, R. C., "The Effects of Free-Stream Turbulence Intensity and Velocity Distribution on Heat Transfer to Curved Surfaces," *Journal of Engineering for Power,* Vol. 100, Jan. 1978, pp. 159–168.

[5]Priddy, W. J., and Bayley, F. J., "Effects of Free Stream Turbulence on the Distribution of Heat Transfer Around Turbine Blade Sections," *International Journal of Heat and Fluid Flow,* Vol. 6, Sept. 1985, pp. 181–192.

[6]Gong, L., Ko, W. L., and Quinn, R. D., "Thermal Response of Space Shuttle Wing During Reentry Heating," AIAA Paper 84-1761, June 1984.

[7]Narasimha, R., "The Laminar-Turbulent Transition Zone in the Boundary Layer," *Progress in Aerospace Sciences,* AIAA, New York, Vol. 22, No. 1, 1985, pp. 29–80.

[8]Dhawan, S., and Narasimha, R., "Some Properties of Boundary Layer Flow During the Transition from Laminar to Turbulent Motion," *Journal of Fluid Mechanics,* Vol. 3, Pt. 4, 1958, pp. 418–436.

[9]Lakshminarayana, L., "Flight Heat Transfer to a Nose Cone," Dept. of Aeronautical Engineering, Indian Institute of Science, Bangalore, Rept. 77 FM 12, Nov. 1977.

[10]Narasimha, R., Devasia, K. J., Gururani, G., and Badrinarayanan, M. A., "Transitional Intermittency in Boundary Layers Subjected to Pressure Gradient," *Experiments in Fluids,* Vol. 2, No. 4, 1984, pp. 171–176.

[11]Blair, M. F., and Werle, M. J., "Combined Influence of Free-Stream Turbulence and Favorable Pressure Gradients on Boundary Layer Transition and Heat Transfer," United Technologies Research Center, CT, Rept. R81-914388-17, March 1981.

[12]Blair, M. F., and Werle, M. J., "The Influence of Free-Stream Turbulence on the Zero Pressure Gradient Fully Turbulent Boundary Layer," United Technologies Research Center, CT, Rept. R80-914388-12, Sept. 1980.

[13]Chen, K. K., and Thyson, N. A., "Extension of Emmons' Spot Theory to Flows on Blunt Bodies," *AIAA Journal,* Vol. 9, No. 5, 1971, pp. 821–825.

[14]Dey, J., and Narasimha, R., "An Integral Method For the Transition Zone in Incompressible Boundary Layers—Part 1," Aerospace Engineering Dept., Indian Institute of Science, Bangalore, Rept. 86 FM 3, June 1986.

[15]Arnal, D., "Three-Dimensional Boundary Layers: Laminar-Turbulent Transition," AGARD-FDP-VKI, April 1986.

[16]Cebeci, T., and Smith, A. M. O., "Transport Coefficients in Turbulent Boundary Layers," *Analysis of Turbulent Boundary Layers,* Academic Press, New York, 1974, pp. 234–239.

[17]Adams, J. C., Jr., "Numerical Calculation of Sharp Flat Plate Transitional and Turbulent Skin Friction," *AIAA Journal,* Vol. 10, 1972, pp. 841–843.

[18]Harris, J. E., "Numerical Solutions of Equations for Compressible Laminar, Transitional and Turbulent Boundary Layers, and Comparisons with Experimental Data," NASA TR R-368 N 71-32164, 1971.

[19]Kuhn, G. D., "Calculation of Compressible, Nonadiabatic Boundary Layers in Laminar, Transitional and Turbulent Flow by the Method of Integral Relations," NASA CR-1797, Nov. 1971.

[20]Gaugler, R. E., "A Review and Analysis of Boundary Layer Transition Data for Turbine Application," NASA TM 86880, March 1985.

[21]Michel, R., Coustols, E., and Arnal, D., "Transition Calculations in Three-dimensional Flows," *Proceedings of the Third Symposium on Numerical and Physical Aspects of Aerodynamic Flows,* edited by T. Cebeci, Springer-Verlag, New York, pp. 39–56.

[22]McDonald, H., and Fish, R. W., "Practical Calculations of Transitional Boundary Layers," *International Journal of Heat and Mass Transfer,* Vol. 16, Sept. 1973, pp. 1729–1744.

[23]Wilcox, D. C., "Alternative to the e^9 Procedure for Predicting Boundary-Layer Transition," *AIAA Journal,* Vol. 19, No. 1, 1981, pp. 56–64.

[24a]Arad, E., Berger, M., Israeli, M., and Wolfshtein, M., "Numerical Calculation of Transitional Boundary Layers," *International Journal for Numerical Methods in Fluids,* Vol. 2, No. 1, 1982, pp. 1–23.

[24b]Arad, E., Israeli, M., and Wolfshtein, M., "The Calculation of Transitional Boundary Layer by a Two-Equation Turbulent Model," *Proceedings of the Eighth Australasain Fluid Mechanics Conference,* edited by R. A. Antonia, Univ. of Newcastle, Newcastle, Australia, 1983.

[25]Vancoillie, G., "A Turbulence Model for the Numerical Simulation of Transitional Boundary Layers," *Second IUTAM Symposium on Laminar-Turbulent Transition,* edited by V. V. Kozlov, Springer-Verlag, New York, pp. 87–92.

[26]Wang, J. H., Jen, H. F., and Hartel, E. O., "Airfoil Heat Transfer Calculation Using a Low Reynolds Number Version of a Two-equation Turbulence Model," *Journal of Engineering for Gas Turbines and Power,* Vol. 107, Jan. 1985, pp. 60–67.

[27]Tanaka, H., Kawamura, H., Tateno, A., and Hatamiya, S., "Effect of Laminarization and Retransition on Heat Transfer for Low Reynolds Number Flow Through a Converging to Constant Area Duct," *Journal of Heat Transfer,* Vol. 104, May 1982, pp. 363–371.

[28]Fraser, C. J., and Milne, J. S., "Integral Calculations of Transitional Boundary Layers," *Proceedings of the Institute of Mechanical Engineers,* Vol. 200, No. C3, 1986, pp. 179–187.

[29]Narasimha, R., "Subtransitions in the Transition Zone," *Proceedings of Second IUTAM Symposium on Laminar-Turbuent Transition,* edited by V. V. Kozlov, Springer-Verlag, New York, pp. 141–151.

[30]Narasimha, R., and Dey, J., "Transitional Spot Formation Rate in Two-dimensional Boundary Layers," *Proceedings of the Third Symposium on Numerical and Physical Aspects of Aerodynamic Flows,* edited by T. Cebeci, Springer-Verlag, New York, pp. 57–74.

[31]Kline, S. J., Morkovin, M. V., Sovran, G., and Cockrell, D. J. (eds.), *Proceedings Computation of Turbulent Boundary Layers—1968 AFOSR-IEP-Stanford Conference,* 1968, Vol. 1, pp. 464–482, Vol. 11, pp. 1–19.

[32]Thwaites, B., "Approximate Calculation of the Laminar Boundary Layer," *Aeronautical Quarterly,* Vol. 1, Nov. 1949, pp. 245–280.

[33]Dey, J., and Narasimha, R., "An Extension of Thwaites's Method for Calculation of Incompressible Laminar Boundary Layers," *Journal of Indian Institute of Science,* Vol. 70, No. 1, 1990, pp. 1–24.

[34]Green, J. E., Weeks, D. J., and Brooman, J. W. F., "Prediction of Turbulent Boundary Layers and Wakes in Compressible Flow by a Lag-Entrainment Method," Royal Aircraft Establishment, Farnborough, Tech. Rept. 72231, Jan. 1973.

[35]Govindarajan, R., and Narasimha, R., "The Role of Residual Non-Turbulent Disturbances On Transition Onset in Two-Dimensional Boundary Layers," National Aeronautical Lab., Bangalore, Rept. TM DU8901, June 1989.

[36]Narasimha, R., "On the Distribution of Intermittency in the Transition Region of a Boundary Layer," *Journal of the Aeronautical Sciences,* Vol. 24, Sept. 1957, pp. 711, 712.

[37]Abu-Ghannam, B. J., and Shaw, R., "Natural Transition of Boundary Layers—the Effects of Turbulence, Pressure Gradient, and Flow History," *Journal of Mechanical Engineering Science,* Vol. 22, Oct. 1980, pp. 213–228.

[38]Owen, F. K., "Transition Experiments on a Flat Plate at Subsonic and Supersonic Speeds," *AIAA Journal,* Vol. 8, No. 3, 1970, pp. 518–523.

[39]Gostelow, J. P., and Blunden, A. R., "Investigation of Boundary Layer Transition in Adverse Pressure Gradient," American Society of Mechanical Engineers, Paper 88-GT-298, Jan. 1988.

[40]Owen, F. K., and Horstman, C. C., "Hypersonic Transitional Boundary Layers," *AIAA Journal,* Vol. 10, No. 6, 1972, pp. 769–775.

[41]Schubauer, G. B., and Klebanoff, P. S., "Contributions on the Mechanics of Boundary-Layer Transition," NACA TN 3489, Sept. 1955.

[42]Dey, J., and Narasimha, R., "Effect of Favourable Pressure Gradient on Transitional Spot Formation Rate," *Journal of Experimental Thermal and Fluid Science* (to be published).

[43]Pohlhausen, K., "Zur Naherungsweisen Integration der Differentialgleichung der Laminaren Reibungsschicht," *Zeitschrift fur Angewandte Mathematik und Mechanik,* Vol. 1, No. 1, 1921, pp. 252–268.

[44]Head, M. R., "Entrainment in the Turbulent Boundary Layer," Aeronautical Research Council R&M 3152, 1958.

[45]Desai, S. S., and Kiske, S., "A Computer Program to Calculate Turbulent Boundary Layers and Wakes in Compressible Flow With Arbitrary Pressure Gradient Based on Green's Lag-Entrainment Method," Institute für Thermo-und-Fluiddynamik, Ruhr University, Bericht Nr. 89, 1982.

[46]Purtell, L. P., Klebanoff, P. S., and Buckley, F. T., "Turbulent Boundary Layer at Low Reynolds Number," *Physics of Fluids,* Vol. 24, May 1981, pp. 802–811.

[47]Dey, J., and Narasimha, R., "An Integral Method For The Calculation of 2-D Transitional Boundary Layers," Aerospace Engineering Dept., Indian Institute of Science, Bangalore, Rept. 88FM7, Dec. 1988.

[48]McDonald, H., and Kreskovsky, J. P., "Effect of Freestream Turbulence on the Turbulent Boundary Layer," *International Journal of Heat and Mass Transfer,* Vol. 17, July 1974, pp. 705–716.

[49]Blair, M. F., private communication.

[50]Gostelow, J. P., "Adverse Pressure Gradient Effects On Boundary Layer Transition in a Turbulent Free Stream," Ninth International Symposisum on Air-Breathing Engines, Athens, Sept. 1989.